

Testing the Equivalence Principle in Quantum Physics

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Abstract

We showed in a previous paper that a wide class of nonmetric theories of gravity encompassed by the χg formalism predict that the speed of light rays depends on the their polarization direction relative to directions singled out by the gravitational field. This gravity-induced birefringence of space is a violation of the Equivalence Principle and is due to the nonmetric coupling between gravity and electromagnetism. In this paper we analyze the propagation of light in the gravitational field of a rotating black hole when nonmetric couplings to curvature are included and compute the time delay between rays with orthogonal polarizations. We obtain an upper bound on the strength of QED like and CP violating curvature couplings using time delay data for pulsar PSR 1937+21. By comparison the corresponding coupling strength for QED coupling is 37 orders of magnitude less.

I. INTRODUCTION

What distinguishes metric from nonmetric theories is the manner in which they couple matter to the gravitational field. Metric theories admit representations in which a single symmetric, second-rank tensor gravitational field couples to matter in a specific way that assures the validity of the Einstein equivalence principle, so-called universal coupling. In contrast, typical nonmetric theories feature additional gravitational fields that also couple directly to matter. For example, Moffat's nonsymmetric gravitation theory (NGT) features an antisymmetric tensor gravitational field that couples to the electromagnetic field. Couplings to fields like this violate the Einstein equivalence principle by causing local nongravitational physics to depend on the additional field values at different events in spacetime. Ni [1] introduced the χg Lagrangian governing electrodynamics in arbitrary gravitational fields governed by any of a broad range of metric and nonmetric theories. The coupling between gravity and electromagnetism is represented by a fourth rank tensor, χ . Symmetries of the electromagnetic Lagrangian imply that only twenty-one components are independent. In this setting, the challenge facing gravitation physicists is to design and perform experiments whose results either force this Lagrangian toward the metric form or reveal the effects of departures from that form.

We begin by recalling results we obtained for classical electromagnetism. A first paper [2] dealt with a phenomenon that is a reflection of spatial anisotropy induced by nonmetric

gravitational couplings to the electromagnetic field. This anisotropy can induce a birefringence of space. Two orthogonal linear polarization states of light are singled out by a nonmetric field and propagate with different phase velocities. This effect was first noticed by Gabriel *et al.* [3] in a particular nonmetric theory, Moffat's NGT. He and his collaborators used observations of polarized light emitted from magnetically active regions near the Sun's limb to impose sharp new constraints on that theory. Our first paper interpreted this new type of equivalence principle test in the broader context of the χg formalism. It was shown that ten of the χg formalism's twenty-on nonmetric degrees of freedom induce birefringence, a fact known to Ni. Also, data on the polarization of radio galaxies were used to impose constraints that are 10^8 times sharper than previous ones [4].

In a second paper [5] we present the first detailed analysis of the constraints which atomic anisotropy experiments impose on all of the nonmetric degrees of freedom encompassed by Ni's formalism. It also establishes that results of Hughes-Drever-like [6] experiments, in concert with results of birefringence experiments, sharply constrain all but two of the nonmetric degrees of freedom encompassed by the χg formalism. The Eötvös experiment constrains one of the remaining two.

In summary, the combined tests of atomic anisotropy and birefringence of space impose constraints on all degrees of freedom representing the coupling of gravity with classical electromagnetism, but one. The sharpness of those constraints strongly supports the validity of the Einstein equivalence principle.

Things change drastically however when the electromagnetic field is treated as a quantum field. Photons can exist as virtual electron positrons pairs. Such pairs have a spatial extension on the order of the Compton wavelength of electron, and are subject to tidal forces due to background curvature of space. This effect goes beyond considerations pertaining to the Einstein Equivalence principle. Local classical theories of matter have no curvature couplings. This particular coupling between curvature and the electromagnetic field can lead to effects that mimic violations of the principle of equivalence because the propagation of photons is directly affected by the existence of preferred directions in space singled out by the curvature of the external gravitational field.

Birefringence and other effects induced by the interaction of a quantum field with a prescribed external field is a phenomenon that has been studied before. Euler and Heisenberg [7] computed the effective action describing the nonlinear interaction of an electromagnetic field with itself through vacuum polarization. This action was obtained subsequently by Schwinger [8] using the background field method he pioneered. Adler [10] used Schwinger's expression to study the quantum propagation of light in an external magnetic field. He found that the local speed of light depends on its polarization direction relative to the orientation of the magnetic field. The effective action governing the propagation of light in a background gravitational field was derived by de Wit, using and extending the background field method. This action is the starting point for studying effects of curvature coupling to electromagnetism.

The breaking of local Lorentz invariance due to curvature coupling causes a background dependence of the local velocity of light, i.e. curvature-induced birefringence. Recently, Drummond and Hathrell [9] studied the propagation of light in the gravitational field of a black hole and in cosmological geometries. For background geometries that are not isotropic, they found photon trajectories and polarizations for which the local speed of light differs

from *c.* Daniels and Shore [11,12] performed a similar analysis for charged and rotating black holes, and found birefringence for radial photon trajectories in the latter case. This can be understood from the fact that the gravitational field of rotating black holes singles out the direction of their axis of rotation.

The coupling of gravity to electromagnetism via curvature is conceptually very different from the couplings considered in previous papers. On the other hand, the effective action describing curvature coupling can be cast into a form that is encompassed by general Ni's Lagrangian introduced above.

The strength of the curvature coupling between QED and gravity is proportional to the ratio of the fine structure constant to the mass of the electron squared. More general curvature couplings of the type exhibited by QED in gravitational background can be considered for which the parameters setting the scale for the strength of the coupling are free. One can use observations to restrict their possible values. Pulses of light of different polarization direction emitted by pulsars reach the Earth with a time delay due to difference in their propagation velocity. Experimental limits on measurement of time of arrival provides a constraint on strength of curvature coupling responsible for birefringence.

When writing down the effective action for curvature coupling between gravity and electromagnetism, one restricts oneself to couplings with the electromagnetic field strength. One could also consider the dual of the field strength. The reason that possibility is discarded is that such coupling violates parity and time reversal. One can however conceive the possibility for quantized gravitational field of coupling to electromagnetism in a way that is not invariant upon space and time inversions. Prasanna and Mohanty [13] introduce parity and time reversal violating coupling between photons and gravitons and compute the time delay between circularly-polarized light pulses propagating radially from a pulsar to Earth. They use data from pulsar PSR 1937+21 to impose upper limit on one parity violating coupling coefficient. Their resulting time delay is linear in angular momentum of the rotating pulsar, instead of being quadratic as for QED like curvature couplings. In addition, they find that the polarization modes singled out by the gravitational field are circular polarization, whereas the study presented in our paper on birefringence clearly shows that these must be linearly polarized.

In this paper we compute the time delay for light rays for QED like and CP violating curvature coupling using Ni's formalism and impose a constraint on parameter governing the strength of the coupling. In practice, the strongest constraint on curvature coupling will come from experiments that search for anisotropy induced by strong gravitational fields. Such fields exist in the vicinity of black holes or pulsars. Light pulses of different polarization propagating away from those compact objects reach the Earth with a time delay due to their different velocity. This time delay is a cumulative effect and existing pulsar data are used to constrain the strength of parameters entering general curvature coupling.

Because anisotropy in the propagation of light is due to the existence of preferred directions selected by the gravitational field, birefringence will be present for rays emanating radially from compact objects whose gravity field is not spherically symmetric. Rotating objects such as pulsars single out the direction of their axis of rotation and are therefore good candidates for birefringence effects that interest us. We obtain the relative velocity difference between light pulses of different polarization for QED like and CP violating coupling. We correct a mistake in Daniels and Shore's computation, which does not affect their

result in a significant way. Our expression in the CP violating case involves the square of the angular momentum, as expected. We then integrate the relative difference in speed of light along a ray emanating from the surface of the pulsar and reaching the Earth. Comparison with data from PSR 1937+21 yields an upper bound on the strength of general curvature coupling.

II. PHOTON PROPAGATION IN KERR GEOMETRY

The equations of motion for the electromagnetic field when subject to general curvature coupling with gravity are obtained from

$$\frac{\delta\Gamma}{\delta A_\mu(x)} = 0 \quad (1)$$

where the effective action Γ decomposes into

$$\Gamma_0 = -\frac{1}{16\pi} \int d^4x \sqrt{-g} F_{\mu\nu} F^{\mu\nu} \quad (2)$$

and Γ_1 represents general curvature coupling of the type exhibited by one-loop QED in a gravitational field

$$\Gamma_1 = -\frac{1}{16\pi} \int d^4x \sqrt{-g} (\alpha R F_{\mu\nu} F^{\mu\nu} + \beta R_{\mu\nu} F^{\mu\lambda} F^\nu{}_\lambda + \sigma R_{\mu\nu\lambda\rho} F^{\mu\nu} F^{\lambda\rho}) \quad (3)$$

The corresponding Lagrangian is of the form

$$\mathcal{L}_{eff} = -\frac{1}{16\pi} \chi^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta} \quad (4)$$

Where $\chi^{\alpha\beta\gamma\delta}$ is expressed in terms of the Riemann tensor and its contractions. We evaluate it for the gravitational field created by a rotating compact object.

The geometry outside a rotating compact object such as a pulsar or black hole, is described in general relativity by the Kerr metric. Even though curvature couplings are outside general relativity, they represent a first order correction and are assumed not to lead to significant modifications to the Kerr solution. The metric outside a pulsar depends on its mass M and angular momentum Ma , expressed in geometrized units where $c = G = 1$. It describes the classical final state of the collapse of a rotating electrically neutral object. One important feature of the Kerr metric is that it contains a non diagonal component $g_{\phi t}$ expressing the dragging of inertial frames at the horizon at the angular velocity

$$\omega = \frac{2aMr}{\Sigma^2} \quad (5)$$

where

$$\Sigma^2 = (r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta \quad \Delta = r^2 - 2Mr + a^2 \quad (6)$$

It will be necessary to compensate for this rotation when introducing a local Lorentz coordinate system.

In analysis of birefringence of light presented in chapter two, we expressed the relative difference in light velocity between different polarizations in function of values of coupling tensor in local quasi-Lorentz frame

$$\frac{\delta c}{c} = \frac{1}{2} \sqrt{(A - C)^2 + 4B^2} \quad (7)$$

where we recall

$$\begin{aligned} A &= \delta\chi^{0303} - 2\delta\chi^{0331} - \delta\chi^{3131} \\ B &= \delta\chi^{0203} + \delta\chi^{0312} + \delta\chi^{0213} + \delta\chi^{3112} \\ C &= \delta\chi^{0202} - 2\delta\chi^{0221} - \delta\chi^{2121} \end{aligned} \quad (8)$$

in a frame where the direction of light propagation is \hat{r} , and

$$\chi^{\alpha\beta\gamma\delta} = \chi_{Lorentz}^{\alpha\beta\gamma\delta} + \delta\chi^{\alpha\beta\gamma\delta} \quad (9)$$

Since outside the black hole the curvature scalar and the Ricci tensor vanish, one has

$$\delta\chi^{\alpha\beta\gamma\delta} = \sqrt{-g}\sigma R^{\alpha\beta\gamma\delta} \quad (10)$$

The coordinate transformation between Kerr and local Lorentz coordinates is

$$\begin{aligned} t' &= t\sqrt{g_{00} - \omega^2 g_{\phi\phi}} \\ \theta' &= \theta\sqrt{g_{\theta\theta}} \\ \phi' &= \sqrt{g_{\phi\phi}}(\phi + \omega t) \\ r' &= r\sqrt{g_{rr}} \end{aligned} \quad (11)$$

The Riemann tensor components we need for radial propagation are [15] [12]

$$\begin{aligned} R_{0101} &= a = \frac{Mr}{\rho^6\Sigma^2}(r^2 - 3a^2 \cos^2\theta)(2(r^2 + a^2)^2 + a^2\Delta \sin^2\theta) \\ R_{0202} &= b = -\frac{Mr}{\rho^6\Sigma^2}(r^2 - 3a^2 \cos^2\theta)((r^2 + a^2)^2 + 2a^2\Delta \sin^2\theta) \\ R_{0123} &= c = -\frac{Ma \cos\theta}{\rho^6\Sigma^2}(3r^2 - a^2 \cos^2\theta)(2(r^2 + a^2)^2 + a^2\Delta \sin^2\theta) \\ R_{0231} &= d = \frac{Ma \cos\theta}{\rho^6\Sigma^2}(3r^2 - a^2 \cos^2\theta)((r^2 + a^2)^2 + 2a^2\Delta \sin^2\theta) \\ R_{2323} &= -R_{0101} \quad R_{1313} = -R_{0202} \quad R_{1212} = -R_{0303} = R_{0101} + R_{0202} \\ R_{0312} &= -R_{0123} - R_{0231} \quad R_{3132} = -R_{0102} \quad R_{0223} = -R_{0113} \end{aligned} \quad (12)$$

Note that, in this local Lorentz frame, indices are raised and lowered using the Minkowski metric.

With the corresponding expression for $\delta\chi$ in the local tetrad, one finds

$$\frac{\delta c}{c} = \sigma\Delta \frac{3Ma^2 \sin^2\theta}{\rho^6\Sigma^2} \sqrt{r^2(r^2 - 3a^2 \cos^2\theta)^2 + a^2 \cos^2\theta(3r^2 - a^2 \cos^2\theta)^2} \quad (13)$$

This expression differs from that obtained by Daniels and Shore, due to their incorrect expression for the Riemann tensor (their Eq. 3.8). The velocity shift vanishes on the horizon for each polarization.

III. BIREFRINGENCE OF LIGHT IN CP VIOLATING INTERACTIONS WITH GRAVITY

When writing down the effective action for quantum electrodynamics in curved background, one restricts oneself to couplings with the electromagnetic field strength. On pure dimensional grounds, one could also consider the dual of the field strength. The reason that possibility is discarded is that such coupling violate parity. On the other hand, the very existence of black hole radiance suggests that gravity might violate T invariance [14]. Lack of time reversal invariance occurs because the evolution from an initial pure state such as the *in* vacuum toward a mixed state characterized by the temperature of the black hole radiation, is irreversible. Prasanna and Mohanty [13] are considering possible CP violating couplings between photons and gravitons. Dimensional analysis in units of mass restricts the form the interaction Lagrangian as follows. Keeping terms quadratic in electromagnetic field strength constraints interaction terms to contain only gravitational expression of mass dimension two. Such objects involve at most two derivatives of the metric. General covariance then forces those to be formed out of local geometrical invariants such as the Riemann tensor and its contractions. The resulting expression is then similar to QED like coupling to gravitational field.

Constructing a Lagrangian describing CP violating graviton-photon coupling is similar to building *ab initio* an effective Lagrangian for one-loop QED in curved spacetime. One writes all gauge invariant and diffeomorphism invariants operators based on field strength and curvature tensor. Those operators can be ordered by increasing mass dimension. In the case of CP violating interaction, the lowest dimensional operator has dimension four and is $F^{\mu\nu}\tilde{F}_{\mu\nu}$. Since it is a total divergence in four dimensional spacetime, it plays no dynamical role in the absence of boundaries. The next terms all have dimension six, thus

$$\mathcal{L}_{int} = -\frac{1}{16\pi}\sqrt{-g}(F_{\mu\nu}F^{\mu\nu} + c_1R_{\mu\nu\alpha\beta}F^{\mu\nu}\tilde{F}^{\alpha\beta} + c_2R_{\mu\nu}F^{\mu\alpha}\tilde{F}^{\nu\alpha} + c_3RF^{\mu\nu}\tilde{F}_{\mu\nu}) \quad (14)$$

where $\tilde{F}_{\mu\nu} = 1/2\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}$ is the dual of F . All coefficients c_i have thus mass dimension minus two. The last six dimensional interaction term involves one derivative of F and \tilde{F} and does not couple to gravity.

Outside the pulsar, both the Ricci tensor and the curvature scalar vanish, and the resulting interaction Lagrangian only involves the indeterminate coefficient c_1 . From the expression for the CP violating Lagrangian one extracts the gravitational tensor $\delta\chi$

$$\delta\chi^{acef} = \frac{c_1}{2}R^{acbd}\epsilon_{bd}^{ef} \quad (15)$$

For this Lagrangian, the relative difference of light velocity is

$$\frac{\delta c}{c} = c_1\Delta\frac{3Ma^2\sin^2\theta}{\rho^6\Sigma^2}\sqrt{r^2(r^2 - 3a^2\cos^2\theta)^2 + a^2\cos^2\theta(3r^2 - a^2\cos^2\theta)^2} \quad (16)$$

The phase shift is thus identical to the one obtained in QED. Note that this expression does vanish on the horizon, just like in the QED case. Therefore the CP violating coupling satisfies the horizon theorem [16] which says that at the event horizon, (where Δ vanishes), the light cone is not modified by the coupling with the background geometry. This theorem holds at the classical level and for one loop QED in curved spacetime.

IV. EXPERIMENTAL CONSTRAINTS

We use time measurements from pulsars to constrain the magnitude of curvature coupling coefficient. For such rotating objects, lowest order contribution in angular momentum parameter a to the velocity shift gives reasonable order of magnitude for comparisons with observations. One finds

$$\frac{\delta c}{c} = c_1 \frac{3Ma^2 \sin^2 \theta}{r^5} \left(1 - \frac{2M}{r}\right) + O\left(\frac{c_1 Ma^4}{r^7}\right) \quad (17)$$

To obtain the integrated phase shift, we must express the relative velocity shift as a coordinate quantity. That is the expression we just derived measures ratios of local Lorentz length and time intervals. The measured time delay at infinity differs from that measured in the vicinity of the black hole. The actual phase shift must thus be multiplied by $\sqrt{\frac{g_{rr}}{-g_{00}}}$. Since the local velocity shift is a second order quantity in a , we only keep the a independent terms in the metric coefficients, i.e we use the Schwarzschild metric. The integrated time delay from the surface of the pulsar to infinity yields

$$\Delta t = \int_R^\infty c_1 \frac{3Ma^2 \sin^2(\theta)}{r^5} \left(1 - \frac{2M}{r}\right)^2 dr \quad (18)$$

To obtain an order of magnitude for the strength of CP violating curvature coupling one can choose an equatorial ray, for which one obtains.

$$\Delta t = \frac{1}{20} c_1 \frac{J^2 G}{c^5 M R^4} \quad (19)$$

The time delay for pulsar PSR 1937+21 is less than $10^{-6}s$. For this pulsar, of angular momentum $J = Mac = 10^{41} kg m^2 s^{-1}$, radius $R = 10 km$ and mass $M = 1.4 M_\odot$ one finds the following constraint for c_1

$$c_1 \leq 10^9 m^2 \quad (20)$$

The same constraint on QED like curvature coupling is obtained, since expression for the relative velocity shift is identical for rays propagating radially.

V. SUMMARY AND CONCLUSIONS

We have derived the time delay of light emitted from pulsars as it travels in a gravitational field when that field couples to electromagnetism through its Riemann tensor. An upper bound on the strength of both QED like and CP violating curvature coupling is obtained by computing the time delay along equatorial rays and using data from pulsar PSR 1937+21. By comparison the corresponding value of σ for QED in curved spacetime is 37 orders of magnitude less.

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